647510 **BSc3MatC0301** **Seat No :________**

B.Sc. Semester - 3 (*CBCS*) Examination

Oct/Nov - 2024 (As Per Syllabus Year June-2017)

MATHEMATICS (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que.1(A) Answer any ONE. (07)

(i) Consider real sequences $\{a_n\}$ and $\{b_n\}$. If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then show that $\lim_{n\to\infty}(a_n b_n) = ab$.

(ii) Prove: Every bounded monotonic sequence is convergent.

Que.1(B) Answer any ONE. (04)

(i) Define a sequence. Prove that the limit of a convergent sequence is unique.

(ii) Define convergence of a sequence. Find limit of sequence $\{x_n\}$, if exist, where $x_{n+1} = \sqrt{6x_n}$ for all $n \in \mathbb{N}$ and $x_1 = \sqrt{6}$.

Que.1(C) Answer any ONE. (03)

(i) Evaluate : $\lim_{n\to\infty} \frac{1}{n}\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right]$.

(ii) Prove that every convergent sequence is bounded.

Que.2(A) Answer any ONE. (07)

(i) Prove: If a real series $\sum u_n$ is convergent then $\lim_{n\to\infty} u_n = 0$, but the converse need not be true.

(ii) State Raabe's test. Discuss the convergence of following series $\sum \frac{1^2 \cdot 4^2 \cdot 7^2 \ldots (3n-2)^2}{3^2 \cdot 6^2 \cdot 9^2 \ldots (3n)^2}$

Que.2(B) Answer any ONE. (04)

(i) State the comparison test. Discuss the convergence of series $\frac{1}{1\cdot2\cdot3} + \frac{3}{2\cdot3\cdot4} + \frac{5}{3\cdot4\cdot5} + \cdots$.

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$.

Que.2(C) Answer any ONE. (03)

(i) Define an alternating series. Also state the Leibnitz test for alternating series.

(ii) Test the convergence of series $\sum_{n=1}^{\infty} \frac{\sqrt{4n+1}}{(n+3)^2}$.

Que.3(A) Answer any ONE. (07)

(i) Define vector point function. If $\bar{f}$ and $\bar{g}$ are vector functions on the domain D whose first order partial derivatives exist, then show that $div\left(\bar{f} \times \bar{g}\right) = \bar{g} \cdot curl\, \bar{f} - \bar{f} \cdot curl\, \bar{g}$.

(ii) Define vector point function. If $\bar{f}$ is a vector function and ϕ is a scalar function such that their first order partial derivatives exist on the domain *D*, then show that $curl\left(\phi \bar{f}\right) = \phi \cdot curl\, \bar{f} + grad\, \phi \times \bar{f}$.

Que.3(B) Answer any ONE. (04)

(i) Define Laplacian operator. Show that the function $f(x,y) = e^{-7x}\,(3\sin 7y + 5\cos 7y)$ satisfies the Laplace equation.

(ii) Define irrotational function. Show that for a scalar function ϕ on domain D, the vector ψ given by $\psi = grad\ \phi$ is irrotational.

Que.3(C) Answer any ONE. (03)

(i) Define solenoidal function. If $\bar{F} = (ax^3z - 2xyz,\ 4xy - x^2yz,\ byz^2 - cxz)$ is solenoidal then find the value of constants $a,\ b,\ c$.

(ii) Find $grad\left(div\,(\bar{F})\right)$, if $\bar{F} = grad\,(x^3 + y^3 + z^3 - x^2y - y^2z - z^2x - 3xyz)$.

Que.4(A) Answer any ONE. (07)

(i) Explain the evaluation of double integral and also explain the change of order of integration.

(ii) Explain the evaluation of triple integral and also explain its transformation from cartesian coordinate system to spherical coordinate system.

Que.4(B) Answer any ONE. (04)

(i) By using the concept of double integral, find the area of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(ii) Using double integral, find the total surface area of sphere $x^2 + y^2 + z^2 = a^2$.

Que.4(C) Answer any ONE. (03)

(i) Change the order of integration of $\int_0^1 \int_{y^2}^{2-y} f(x,y)\ dx\, dy$.

(ii) Evaluate : $\int_0^1 \int_0^{x^2} \int_0^{x+y} z\, dz\ dy\, dx$.

Que.5(A) Answer any ONE. (07)

(i) State and prove the Green's theorem.

(ii) Write the definition of beta function and gamma function. Prove that $\beta(m,n) = \frac{\Gamma(m)\ \Gamma(n)}{\Gamma(m+n)}$ for $m,n > 0$.

Que.5(B) Answer any ONE. (04)

(i) Write the statement of Gauss divergence theorem and Stoke's theorem.

(ii) Evaluate : $\int_0^{\infty} \frac{x^8\,(1-x^6)}{(1+x)^{24}}\, dx$.

Que.5(C) Answer any ONE. (03)

(i) For $\bar{V} = (x^2 + y^2,\ -2xy,\ 0)$ and C as $x^2 + y^2 = 1$, use the Stoke's theorem to find $\oint_C \bar{V}\, ds$.

(ii) Prove : $n\ \beta(m+1,n) = m\ \beta(m,n+1)$ for $m,\ n > 0$.

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